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1994 J. Phys.: Condens. Matter 6 9981

(http://iopscience.iop.org/0953-8984/6/46/016)

Superconducting fluctuation conductivity in granular Al–Ge films above the metal–insulator transition

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Received 4 May 1994, in final form 27 June 1994

Abstract. Superconducting fluctuation conductivities and magnetoconductances have been measured in thin granular Al–Ge films above the superconducting transition temperature. For very metallic films located well above the metal–insulator transition, the two-dimensional Aslamazov–Larkin and Maki–Thompson theories describe the conductivity and magnetoconductance data well. The Maki–Thompson expressions involve the temperature-dependent pair-breaking parameter δ. For films located near the metal–insulator transition, we have observed a dimensional crossover from three dimensions to two dimensions to a fractal dimension with decreasing temperatures. Values for the diffusion constant, ranging from 0.4 to 1.4 $cm^2\ s^{-1}$, were obtained from the magnetoconductance fits, and these magnitudes compared favourably with values obtained from critical-field measurements taken below T_c.

1. Introduction

Superconducting fluctuations have probably been observed for over 40 years now since the pioneering experiments on amorphous bismuth films by Buchel and Hilsch [1] and later by Shier and Ginsberg [2]. Superconducting fluctuation conductivity (SCF) causes the 'rounding' of the resistance transition curve above the superconducting transition temperature in high-resistance thin films. This 'rounding', illustrated in figure 1 for our 2000 Å Al–Ge films having different Al concentrations, is to be contrasted to the sharp, almost discontinuous, transitions observed in bulk superconductors. However, it was Glover who demonstrated that the superconducting fluctuation conductivity, also known as the excess conductivity or paraconductivity, follows a Curie–Weiss type law [3]. The rounding is caused by fluctuations of the superconducting order parameter. Even above T_c, the fluctuations create superconducting Cooper pairs of finite lifetime that contribute to the conductivity. The Curie–Weiss type conductivity law was derived microscopically by Aslamazov and Larkin [4]. Many theoretical and experimental papers followed on the subject, including the early review papers by Glover [5] and by Skocpol and Tinkham [6, 7] and the important Proceedings of the 1969 Stanford International Conference on the Science of Superconductivity [8].

After introducing the relevant and important theoretical formulae, we summarize measurements on the SCF conductivity and magnetoconductivity in granular Al–Ge films located above the metal–insulator transition. The experimental results are in excellent agreement with the theories for the most metallic films. However, for the films located closest to the metal–insulator transition, the classical theories are inadequate to explain the conductivity and magnetoconductance results.

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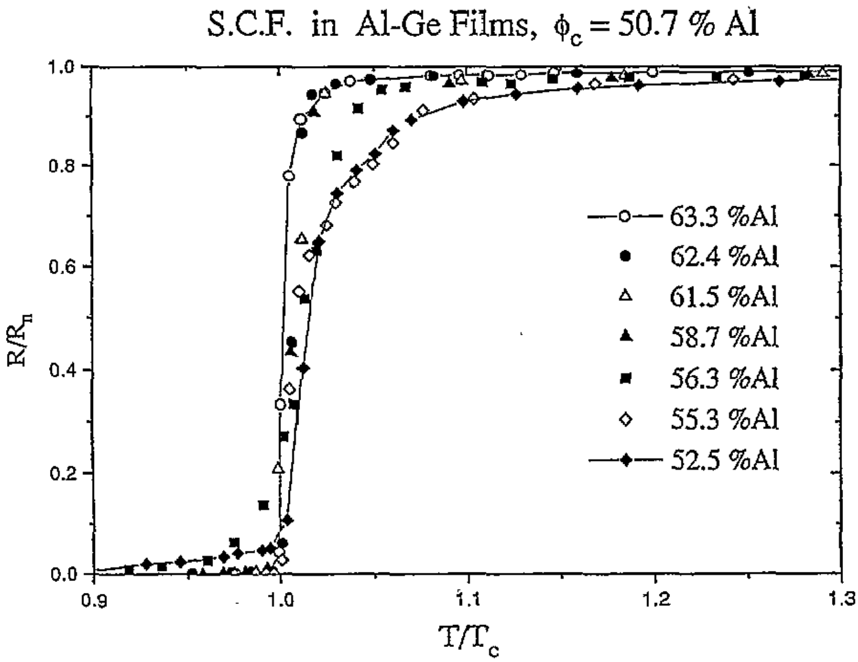

Figure 1. Resistance behaviour in thin 2000 Å Al–Ge films near the superconducting transition temperature. The 'rounding' is due to the presence of superconducting fluctuations above T_c.

2. Theoretical background

2.1. Aslamazov–Larkin theory

About 25 years ago, Aslamazov and Larkin (AL) calculated the influence of superconducting fluctuations on the electrical conductivity above the superconducting transition temperature T_c [4, 9]. The AL contribution results from the direct acceleration of the fluctuation-induced Cooper pairs above T_c. Close to T_c for a two dimensional (2D) film, Aslamazov and Larkin obtained the zero-field expression [4, 9]

$$\sigma_{\square}^{\mathrm{AL,2D}}(T) = e^2/[16\hbar\epsilon(T)] \tag{1}$$

where $\epsilon(T) = T/T_c - 1$. The superconducting fluctuation conductivity $\sigma_{\square}^{\mathrm{SCF}}$ is defined as

$$\sigma_{\square}^{\mathrm{SCF}} = 1/R_{\square}(B = 0, T) - 1/R_{\square}^{n}(T)$$

where $R_{\square}(B = 0, T)$ is the zero magnetic field resistance per square and $R_{\square}^{n}(T)$ is the normal resistance per square measured in a sufficiently strong magnetic field that quenches all superconductivity in the film. Note that the 2D AL expression has no dependence upon the material properties of the film. If one wishes to extend the temperature range of equation (1) considerably above T_c, then one replaces $\epsilon(T) = T/T_c - 1$ by $\ln(T/T_c)$, yielding [3, 10]

$$\sigma_{\square}^{\mathrm{AL,2D}}(T) = e^2/[16\hbar \ln(T/T_c)]. \tag{2}$$

Equations (1) and (2) have been well confirmed in disordered amorphous films by Glover [3].

The 2D Aslamazov–Larkin conductivity expression should be contrasted with the 3D and 1D conductivity expressions in zero field [3]:

$$\sigma^{\mathrm{AL,3D}}(T) = e^2/[32\hbar\xi(0)\epsilon^{1/2}] \tag{3}$$

and

$$\sigma^{\mathrm{AL,1D}}(T) = e^2\pi\xi(0)/[16\hbar S\epsilon^{3/2}] \tag{4}$$

where S is the cross-sectional area of the thin wire and $\xi(0)$ is the zero-temperature coherence length appearing in the Ginzburg–Landau superconductivity coherence length $\xi_{\mathrm{GL}}(T)$:

$$\xi_{\mathrm{GL}}(T) = \xi(0)/|\epsilon(T)|^{1/2} = 0.85(\xi_{\mathrm{BCS}}l)^{1/2}/|\epsilon(T)|^{1/2}. \tag{5}$$

Here ξ_{BCS} is the Bardeen–Cooper–Schrieffer (BCS) coherence length given by $\xi_{\mathrm{BCS}} = 0.18\hbar v_{\mathrm{F}}/k_{\mathrm{B}}T_{\mathrm{c}}$ and l is the elastic mean free path. The 1D and 3D conductivity formulae are material-dependent via $\xi(0)$. Note that the temperature dependence of the 1D expression is much stronger than that of the 3D expression.

The criterion for using the 2D expression is that the superconducting coherence length, ξ_{GL}, must be much greater than the sample thickness, d. For bulk Al, ξ_{BCS} is about 16 000 Å. The BCS coherence length ξ_{BCS} takes on a somewhat smaller value of 12 000 Å owing to the enhanced transition temperature of 1.6 K in our Al–Ge films as compared to the $T_{\mathrm{c}} = 1.19$ K for Al. Assuming that the elastic mean free path l is limited to the typical Al grain diameter of about 100 Å above the metal–insulator transition, then $\xi(0)$ takes on a typical value of 1000 Å. The dependence of the coherence length upon temperature is illustrated in figure 2. Figure 2 suggests that the most metallic films should be best characterized by the 2D formulae only. Moreover, we anticipate that the 2D conduction expressions will be valid for all the films provided that the measurements are made below $1.3T_{\mathrm{c}}$. Near T_{c}, the 2D criterion is well satisfied since, as is illustrated in figure 1, $\xi_{\mathrm{GL}} \to \infty$.

For films located slightly above the metal–insulator transition (MIT), we might anticipate a structural crossover of the Al from 2D clusters and grains to 1D wires and filaments. Thus, we predict that a fractal dimensionality might best describe the Al–Ge excess conductivity close to the MIT. As Entin-Wohlman *et al* [11] first pointed out, and more recently Char and Kapitulnik [12] reconfirmed, the divergence of the AL term is stronger when the 2D expression of equation (1) is replaced by a fractal dimensionality. For percolating clusters, the fractal dimensionality D_{fr} is 4/3, yielding [13]

$$\sigma_{\square}^{\mathrm{AL,fractal}} = e^2/[16\hbar\epsilon^x(T)] \qquad \text{with } x \simeq 1.33. \tag{6}$$

We will show that the fractal expression gives an excellent fit to our conductivity data for films located slightly above the MIT. Actually, the fractal expression (6) is an approximation to the theory. According to Char and Kapitulnik, the total conductivity for each process is the weighted average of two terms [12]: one contribution comes from the homogeneous region and the second contribution comes from the self-similar region. Thus, the Aslamazov–Larkin contribution is composed of two terms, and equation (6) is an approximation to this weighted sum. Likewise, the Maki–Thompson contribution is composed of the weighted sum of two other terms. According to Char and Kapitulnik, the fractal dimension D_{fr} can vary between $1 \leqslant D_{\mathrm{fr}} \leqslant 1.5$, yielding values for x between 1.25 and 1.50 [12].

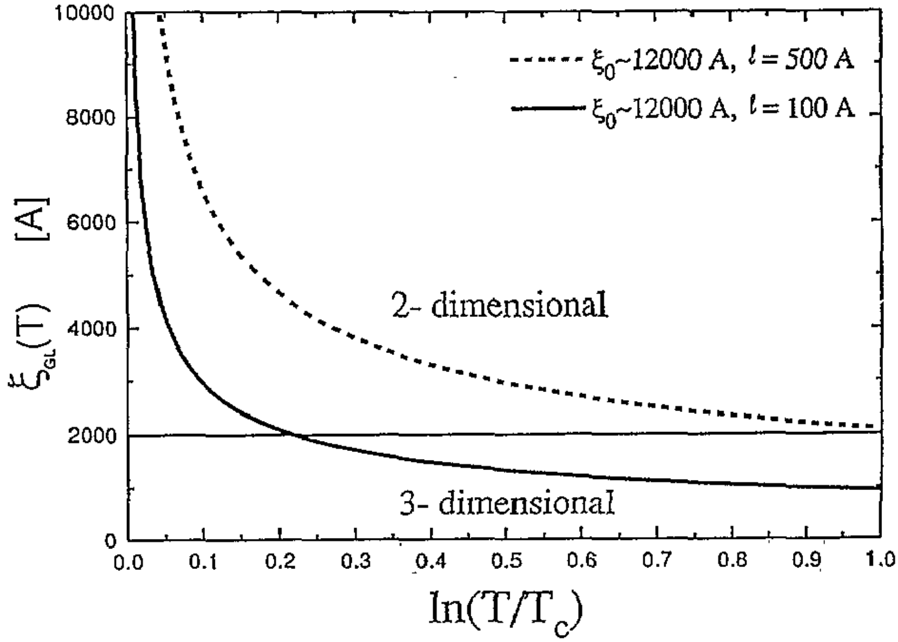

Figure 2. Dependence of the Ginzburg–Landau superconducting coherence length as a function of temperature. The broken curve, which should apply to a very metallic film, suggest 2D behaviour over the entire temperature range, provided that the film thickness is 2000 Å. In contrast, a crossover from 3D to 2D behaviour should be observed in a film located just above the metal–insulator transition.

For highly disordered films exhibiting superconducting fluctuations dominated by the Aslamazov–Larkin term, the magnetoconductance (MC) measurements made in an applied perpendicular field allow a direct determination of the diffusion constant D_{dif} of the films. The magnetoconductance $\Delta\sigma_{\square}$ is defined as

$$\Delta\sigma_{\square}^{\mathrm{SCF,2D}}(B,T) = 1/R_{\square}(B,T) - 1/R_{\square}(B=0,T).$$

Using a microscopic calculation, Redi reconfirmed an expression for the magnetoconductance first derived by Abrahams *et al* using a phenomenological approach [14, 15]:

$$\Delta\sigma_{\square}^{\mathrm{AL,2D}}(B,T) = \sigma_{\square}^{\mathrm{AL,2D}}(B=0,T)\{8z^2[\psi(1/2+z) - \psi(1+z) + 1/2z] - 1\} \tag{7}$$

where z is a fitting parameter strongly dependent upon temperature and given by

$$z = B_{\mathrm{dif}}/B = 2\epsilon(T)k_BT/(\pi e D_{\mathrm{dif}}B) \qquad \text{with } B_{\mathrm{dif}} = 2\epsilon(T)k_BT/(\pi e D_{\mathrm{dif}}). \tag{8}$$

In equation (7), ψ is the digamma function and $\sigma_{\square}^{\mathrm{AL,2D}}(B=0,T)$ is the zero-field AL expression of (1). Equation (7) is valid for temperatures close to T_c; we have extended the Redi MC expression of (7) to higher temperatures by replacing $\epsilon(T) = T/T_c - 1$ by $\ln(T/T_c)$ in equations (1) and (8).

The Redi expression of (7) has two nice properties in that (i) the negative MC arising from superconducting fluctuations displays a quadratic field dependence at low fields:

$$\Delta\sigma_{\square}^{\mathrm{AL,2D}}(B,T) = -\sigma_{\square}^{\mathrm{AL,2D}}(B=0,T)B^2/(8B_{\mathrm{dif}}^2) \tag{9}$$

and (ii) at high fields, the conductivity approaches the normal conductivity as $1/B$ with the MC saturating at the value

$$\Delta\sigma_{\square}^{\mathrm{AL,2D}}(B,T) = -\sigma_{\square}^{\mathrm{AL,2D}}(B=0,T) + ek_{\mathrm{B}}T/(2\pi D_{\mathrm{dif}}\hbar B). \tag{10}$$

Equation (7) has been confirmed experimentally by Serin *et al* [16] and more recently by Denhoff and Gygax [17].

The diffusion constant D_{dif} can be extracted from the parameter z by fitting equation (7) to the MC data. Alternatively, D_{dif} can also be obtained from critical-field measurements taken just below T_{c} [10, 18]:

$$D_{\mathrm{dif}} = -4k_{\mathrm{B}}/(\pi e|\mathrm{d}B/\mathrm{d}T|_{T_{\mathrm{c}}}). \tag{11}$$

According to Entin-Wohlman *et al*, the diffusion constant in a percolating system is much smaller than the diffusion constant in a homogeneous system [11]. Near the MIT, the diffusion constant scales as $D_{\mathrm{dif}} \propto \xi_{\mathrm{p}}^{-\theta} \propto (\phi-\phi_{\mathrm{c}})^{t-\beta}$, where ϕ is the metal fraction in the film, ϕ_{c} is the critical metallic fraction at the MIT, t is the conductivity exponent equal to 1.75 in three dimensions, and β is the finite cluster mass exponent equal to 0.41 [19]. Wind *et al* have reported diffusion constants of the order of 40 to 50 cm^2 s^{-1} in clean Al wires and thin Al films [20]. For a percolating-type film located 3% above the MIT, we estimate a small diffusion constant of the order of 0.5 cm^2 s^{-1}.

2.2. *Maki–Thompson theory*

The Aslamazov–Larkin theory provided excellent agreement with measurements on thin amorphous films having high normal resistances per square. However, measurements on clean Al films by the Brookhaven National Laboratory group and by the University of Rochester group showed superconducting fluctuation conductivity values much larger than the AL predictions [21–24]. Maki suggested another contribution to explain this anomalously large conductivity [25]. The Maki–Thompson (MT) contribution originates from the inertia of the superconducting pairs after decaying into pairs of quasiparticles with opposite momenta. Since elastic scattering by impurity potentials conserves time-reversal symmetry, these quasiparticle pairs continue to have nearly zero total momentum and to produce excess conductivity. The quasiparticle pair lifetime is limited by inelastic scattering, which breaks the quasiparticle pairs. Thus, the more disordered the film, the shorter will be the lifetime and hence the less important becomes the Maki–Thompson contribution. In 1D and 2D, the Maki term gave an infinite conductivity at all temperatures above T_{c}; Thompson showed that the non-physical divergence is prevented by the presence of any pair-breaking effect such as magnetic impurities or a magnetic field [26]. Close to T_{c}, the Maki-Thompson (MT) contribution in zero field for 2D is given by

$$\sigma_{\square}^{\mathrm{MT,2D}}(T) = e^2\ln[\epsilon(T)/\delta]/\{8\hbar[\epsilon(T)-\delta]\} \tag{12}$$

where δ is the pair-breaking parameter given by [27, 28]

$$\delta = \pi\hbar/[8k_{\mathrm{B}}T\tau_{\mathrm{in}}(T)]. \tag{13}$$

Here, τ_{in} is the total inelastic scattering time. For temperatures considerably greater than T_{c}, $\epsilon(T) = T/T_{\mathrm{c}}-1$ should be replaced by $\ln(T/T_{\mathrm{c}})$, thus resulting in a MT excess conductivity of

$$\sigma_{\square}^{\mathrm{MT,2D}}(T) = e^2\ln[\ln(T/T_{\mathrm{c}})/\delta]/\{8\hbar[\ln(T/T_{\mathrm{c}})-\delta]\}. \tag{14}$$

Close to T_c ($\ln(T/T_c) \ll \delta$), the MT contribution is smaller than the AL contribution since the AL term diverges more quickly as $\epsilon^{-1}(T)$ while the MT term diverges more slowly as $\ln\epsilon(T)$. At temperatures considerably above T_c the MT term generally dominates over the AL term provided that the pair-breaking parameter δ is small ($0.001 \leqslant \delta \leqslant 0.01$); the reason for this is that the $\ln[\ln(T/T_c)/\delta]$ term appearing in (14) contributes a sizable factor. Note that when $\epsilon = \delta$, the MT term of (12) is well behaved since the $\epsilon - \delta$ term in the denominator approaches zero at the same rate as the $\ln(\epsilon/\delta)$ in the numerator. For ϵ smaller than δ, both the $\epsilon - \delta$ and the $\ln(\epsilon/\delta)$ terms are negative, thus yielding positive, well defined values for the excess conductivity.

For the case of 3D, the Maki-Thompson conductivity term takes on the form [12]

$$\sigma^{\mathrm{MT,3D}}(T) = e^2/[8\hbar\xi(0)(\epsilon^{1/2} + \delta^{1/2})]. \tag{15}$$

There are several methods to estimate the magnitude of the pair-breaking parameter δ appearing in the MT contribution. From the experimental approach, one can use Thompson's definition of δ in terms of a depressed superconducting transition temperature T_c for the more disordered films [29]:

$$\delta = T_{c0}/T_c - 1 \tag{16}$$

where T_{c0}, a fitting parameter, is the transition temperature for a relatively clean film ($T_{c0} \simeq 1.65$ K in our case). Alternatively, one can use the experimental data of δ for Al films reported by Kajimura and Mikoshiba [30] and also by Crow *et al* [22] to predict approximate δ values for our Al–Ge films:

$$\delta = 3 \times 10^{-4} + 6 \times 10^{-4} R_\square. \tag{17}$$

Theoretical support for equation (17) has been forwarded by Ebisawa *et al* [27].

In addition, one can estimate the pair-breaking parameter δ via (13) using theoretical expressions for the various different inelastic scattering times, as was demonstrated by Gordon and Goldman [31].

There are three important scattering rates in the liquid-helium temperature region. One rate is the electron–phonon rate, $1/\tau_{\mathrm{e-ph}}$, which has been calculated for Al films by Lawrence and Meador [32]:

$$1/\tau_{\mathrm{e-ph}} = 1.6 \times 10^7\ \mathrm{T}^3\ \mathrm{s}^{-1}\ \mathrm{K}^{-3}. \tag{18}$$

Below 2 K, the electron–phonon scattering becomes weak and this term may be neglected.

For inelastic electron-electron scattering in the 2D dirty limit, Altshuler *et al* [33] and Fukuyama and Abrahams [34] have shown that

$$1/\tau_{\mathrm{e-e}} = (k_B T/2\hbar)\left(\frac{R_\square}{\pi\hbar/e^2}\right)\ln\left(\frac{\pi\hbar/e^2}{R_\square}\right). \tag{19}$$

At temperatures very near to T_c, Brenig *et al* have suggested that the presence of superconducting fluctuations is expected to affect the inelastic scatter rate [35, 36]. This rate arises from the inelastic processes associated with the recombination of the electrons into superconducting pairs:

$$1/\tau_{\mathrm{e-fl}} = (k_B T/2\hbar)\left(\frac{R_\square}{\pi\hbar/e^2}\right)\frac{2\ln 2}{\ln(T/T_c) + C} \tag{20}$$

where

$$C = 4\ln 2/[\![\{[\ln(\pi\hbar/e^2R_\square)]^2 + 128\hbar/e^2R_\square\}^{1/2} - \ln(\pi\hbar/e^2R_\square)]\!]. \tag{21}$$

We have made the approximation in estimating the constant C in (21) that our films are quasi-2D films where the elastic mean free path $l \ll d$ and $L_{\mathrm{in}}(T) = [D_{\mathrm{dif}}\tau_{\mathrm{in}}(T)]^{1/2} \gg d$; d is the film thickness and D_{dif} is the diffusion constant. In this case $\epsilon_{\mathrm{F}}\tau/\hbar$ has been replaced by $\pi\hbar/e^2R_\square$. The electron fluctuation rate becomes significant only for T values very close to T_c owing to the $[\ln(T/T_c)+C]$ term appearing in the dominator of (20). For a low-resistance film of 17 $\Omega/\square$, C is small and of the order of 0.02. For temperatures much greater than T_c, the $\ln(T/T_c)$ term is large, yielding a small value for $1/\tau_{\mathrm{e-fl}}$. However, as $T \to T_c$, $\ln(T/T_c) \to 0$; and equation (20) yields an electron–fluctuation rate that exceeds the electron–electron scattering rate by a factor of 10. Gordon and colleagues were the first to detect the $1/\tau_{\mathrm{e-fl}}$ rate [37, 31].

The total rate is the sum of these three scattering rates.

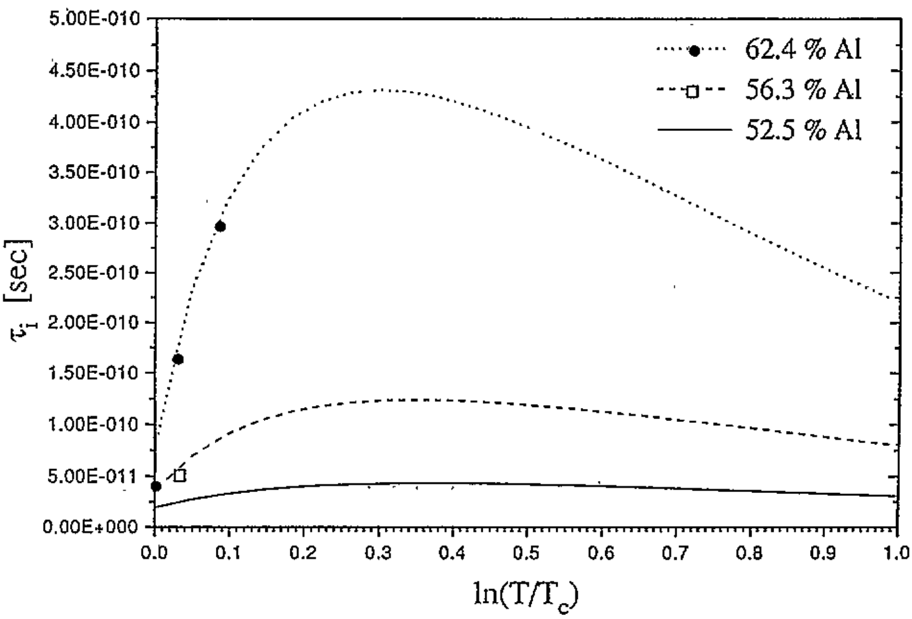

Figure 3. Theoretical predicted inelastic scattering time τ_{in} as a function of temperature for three different film resistances of 17.3, 78.3 and 329 $\Omega/\square$, according to equations (18), (19) and (20). The anomalous decrease of τ_{in} below $\ln(T/T_c)$ of 0.2 ($T < 1.2T_c$) arises from electrons scattering off of Cooper pairs, as suggested by Brenig *et al* [35, 36].

The inelastic time τ_{in}, which includes the above three scattering mechanisms, is illustrated in figure 3 for zero applied field. Here as the temperature decreases, τ_{in} increases, indicating weaker scattering. However, close to T_c ($T < 1.3T_c$), the Brenig expression becomes increasingly important, producing the anomalous decrease in the magnitude of the inelastic time [35, 36]. As $T \to T_c$, the scattering effect increases strongly as the electrons suffer additional inelastic scattering by exchange of superconducting fluctuations. The effect saturates at T_c as more and more electrons condense into Cooper pairs, leaving fewer electrons to participate in the scattering process.

Interestingly, if τ_{e-e} is the dominating time, then according to (13) the pair-breaking parameter δ becomes temperature-independent and scales directly proportional to $R_\Box$, as was observed experimentally in equation (17). The temperature dependence of the pair-breaking parameter δ is shown in figure 4. The δ values predicted from experimental results of equation (17) are indicated by the arrows in figure 4. The theoretical prefactor in front of $R_\Box$ is of the order of 1×10^{-4} using (13) rather than 6×10^{-4}, which is observed experimentally [22, 30]. It is not clear to us what is the reason for this disagreement. According to equation (17), if the film exhibits a $R_\Box$ of 500 $\Omega/\Box$ or greater, then δ takes on such large values ($\delta \simeq 0.3$) that the MT term can be neglected compared to the AL term in such high-resistance disordered films.

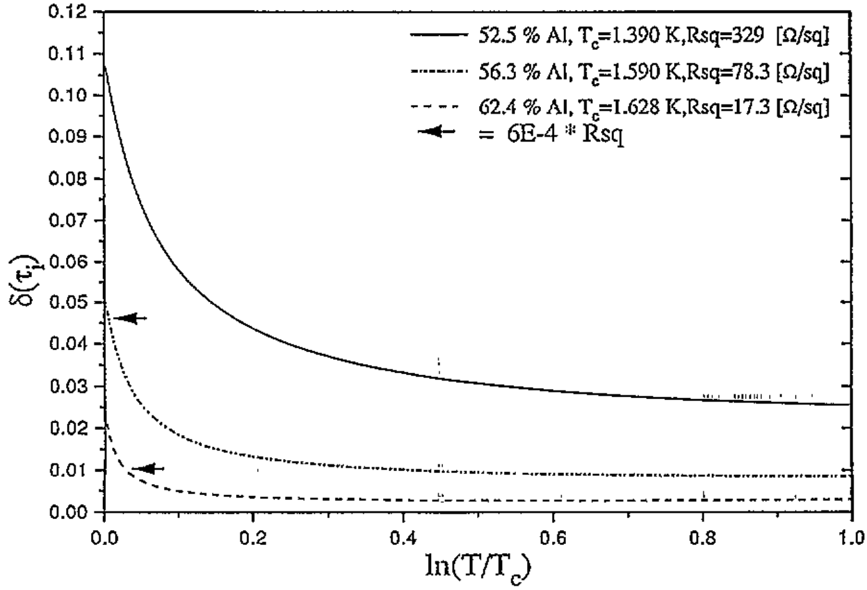

Figure 4. The pair-breaking parameters as a function of temperature. The smaller the magnitude of δ, the more important is the Maki–Thompson contribution to superconducting fluctuations. Note that the pair-breaking parameter is almost temperature-independent, provided that the measuring temperature is not too close to T_c. δ is inversely related to the inelastic scattering time τ_{in}.

Important theoretical work on the 2D Maki-Thompson magnetoconductance (MC) was published by Larkin, who showed that values for the inelastic scattering time $\tau_{in}(T)$ could be deduced from the MT MC data [38]. For small magnetic fields, $\ln(T/T_c) \gg 4eD_{dif}B/k_BT$, and temperatures not too close to T_c, $\ln(T/T_c) \gg \hbar/k_BT\tau_{in}(T)$, Larkin suggested the following expression for the MC [38]:

$$\Delta\sigma_\Box^{MT,2D}(B,T) = -(e^2/2\pi^2\hbar)\beta_L(T)\{\psi[1/2 + B_{in}(T)/B] + \ln[B/B_{in}(T)]\} \tag{22}$$

where $B_{in}(T) = \hbar/[4eD_{dif}\tau_{in}(T)]$ and $\beta_L(T) \simeq \pi^2/[4\ln(T/T_c)]$ for $T \simeq T_c$. For temperatures much greater than T_c, $\beta_L(T) \simeq \pi^2/\{6[\ln(T/T_c)]^2\}$. Parameter $\beta_L(T)$ is known as the Larkin electron–electron interaction strength parameter. For very small fields, equation (22) simplifies to

$$\Delta\sigma_\Box^{MT,2D}(B,T) = -(e^2/2\pi^2\hbar)\beta_L(T)B^2/24B_{in}^2(T) \tag{23}$$

and for stronger fields

$$\Delta\sigma_{\square}^{\mathrm{MT,2D}}(B,T) = -(e^2/2\pi^2\hbar)\beta_{\mathrm{L}}(T)\{\ln[B/B_{\mathrm{in}}(T)] - 1.96\}. \tag{24}$$

Unfortunately, the Larkin expression of equation (24) does not saturate at large fields to the value $\Delta\sigma_{\square}^{\mathrm{MT,2D}}(B \to \infty, T) \simeq -\sigma_{\square}^{\mathrm{MT,2D}}(B = 0, T)$ given by (14). This problem was resolved by Lopes dos Santos and Abrahams [28], who suggested that for temperatures close to $T_{\mathrm{c}}(\ln(T/T_{\mathrm{c}}) \ll 1)$ and for moderate fields such that $B \ll k_{\mathrm{B}}T/4eD_{\mathrm{dif}}$, equation (22) should be replaced by the following expression:

$$\begin{aligned}\Delta\sigma_{\square}^{\mathrm{MT,2D}}(B,T) = &-(e^2/2\pi^2\hbar)\beta_{\mathrm{LS,A}}(T,\delta)\{\psi[1/2 + B_{\mathrm{in}}(T)/B] \\ &- \psi[1/2 + B_{\mathrm{dif}}(T)/B] + \ln(B_{\mathrm{dif}}/B_{\mathrm{in}})\}\end{aligned} \tag{25}$$

where $\beta_{\mathrm{LS,A}}(T,\delta)$ differs slightly from Larkin's $\beta_{\mathrm{L}} = \pi^2/[4\ln(T/T_{\mathrm{c}})]$, being

$$\beta_{\mathrm{LS,A}}(T,\delta) = \pi^2/\{4[\ln(T/T_{\mathrm{c}}) - \delta]\} \qquad \text{for } T \simeq T_{\mathrm{c}}. \tag{26}$$

Also in equation (25), $B_{\mathrm{dif}} = 2\epsilon(T)k_{\mathrm{B}}T/(\pi e D_{\mathrm{dif}}) = B_{\mathrm{in}}\epsilon/\delta$.

Equation (25) has the nice property that at large fields the MC saturates at $-\sigma_{\square}^{\mathrm{MT,2D}}(B = 0, T)$ going as $1/B$:

$$\Delta\sigma_{\square}^{\mathrm{MT,2D}}(B,T) = -\sigma_{\square}^{\mathrm{MT,2D}}(B=0,T) + e^2\pi k_{\mathrm{B}}T/(8\hbar e D_{\mathrm{dif}}B). \tag{27}$$

We have combined (27) with (10) to characterize the high-field asymptotic behaviour of the MC arising from the superconducting fluctuations very near T_{c}. Note that the diffusion constant can be obtained directly by fitting the high-field MC data to this sum:

$$\Delta\sigma_{\square}^{\mathrm{SCF,2D}}(B,T) = \frac{ek_{\mathrm{B}}T}{\hbar D_{\mathrm{dif}}B}\left(\frac{1}{2\pi} + \frac{\pi}{8}\right) - \sigma_{\square}^{\mathrm{SCF,2D}}(B=0,T). \tag{28}$$

Equation (25) also has the desirable property that, for very small fields, it takes on the limiting form of

$$\Delta\sigma_{\square}^{\mathrm{MT,2D}}(B,T) = -\frac{\beta_{\mathrm{L}}(T)e^2}{2\pi^2\hbar}\left(\frac{\ln(T/T_{\mathrm{c}}) - \delta}{\ln(T/T_{\mathrm{c}})}\right)\frac{B^2}{24B_{\mathrm{in}}^2(T)} \tag{29}$$

which is almost identical to the Larkin low-field limit given by (23). Note that the Lopes dos Santos–Abrahams expression underestimates the MC at temperatures much greater than T_{c} and the Larkin expression (22) should be used instead. Gordon and Goldman [31] have successfully used equation (25).

Maki and Thompson have suggested alternative expressions for the MC [39].

2.3. Weak localization and electron–electron interaction theory

At temperatures much greater than T_{c}, there will be a competition between the superconducting fluctuation (SCF) process, which causes a decrease or 'rounding' in the resistance of the film with decreasing temperatures, and the weak localization (WL) and electron–electron interaction (EEI) processes, which cause an increase in the resistance of the film with decreasing temperatures. The zero-field resistance measurements are the sum of these three processes plus the normal resistance. The SCF or excess conductivity is greater in magnitude than the values actually measured. The WL and EEI contributions must be subtracted out in order to avoid large errors in determining the phase-breaking parameter δ. Near T_c, the WL and EEI contributions are negligible; but at 'high' T, from $1.3T_c$ to $3T_c$, these two contributions become important; and at very high temperatures, they completely dominate the conductivity.

According to Lee and Ramakrishnan, the 2D WL contribution for the case of weak spin–orbit scattering can be approximated as [40]

$$\sigma_{\square}^{\mathrm{WL,2D}}(T) = (e^2/2\pi^2\hbar)\,p\ln T \tag{30}$$

where p is the exponent of the temperature term in the most important inelastic scattering time process ($p = 3$ for the electron–phonon scattering time). For the 2D EEI interactions contribution [40]

$$\sigma_{\square}^{\mathrm{EEI,2D}}(T) = (e^2/2\pi^2\hbar)(1 - 3\tilde{F}_{\sigma}/4)\ln T \tag{31}$$

where $\tilde{F}_{\sigma} \simeq 0.2$ is an effective electron screening constant. The sum of the above two terms is about $4(e^2/2\pi^2\hbar)\ln T \simeq 0.000\,049\ln T\,(\Omega/\square)^{-1}$. Failure to subtract out the WL and EEI corrections from the zero-filed data results in values for the Maki–Thompson term that are too small; hence, one obtains unreasonably large values for the pair-breaking parameter, δ.

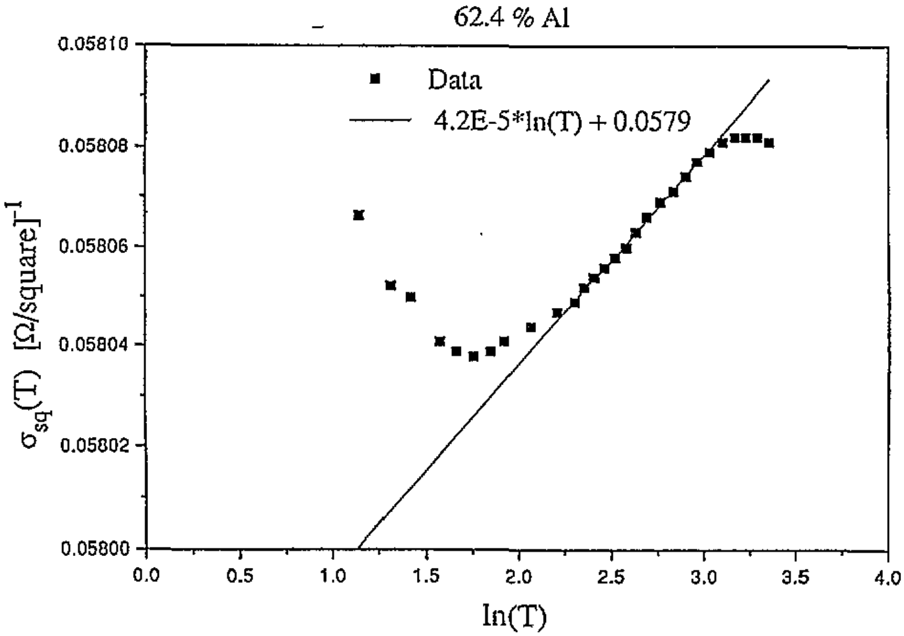

Figure 5. The zero-field conductivity of the 62.4% Al film as a function of temperature, between 3 and 25 K. Between 10 and 20 K, the conductivity is dominated by weak localization and electron–electron interaction effects, represented by the full line. These two quantum corrections have been subtracted out from the zero-field data. The differences between the data points and the extrapolated line represent the SCF data.

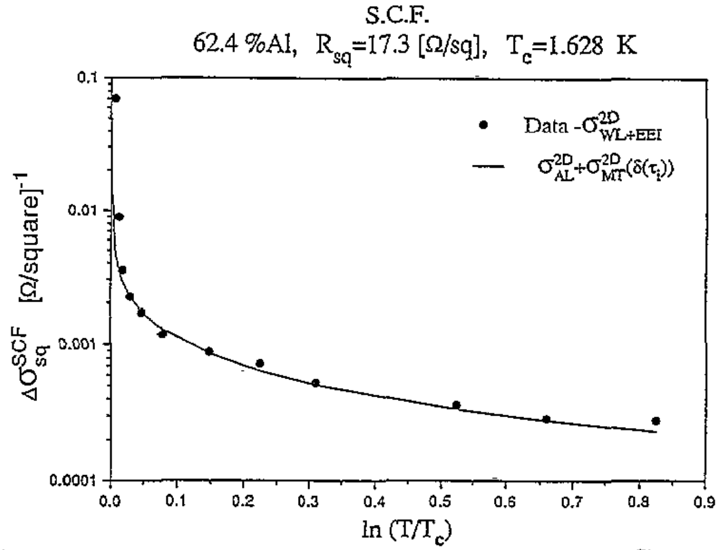

Figure 6. The SCF or excess conductivity data versus temperature for the 'most' metallic film, the 62.4% Al film. The 2D Aslamazov–Larkin and Maki–Thompson expressions fit the data extremely well, provided that the theoretical expressions for the inelastic scattering times are used.

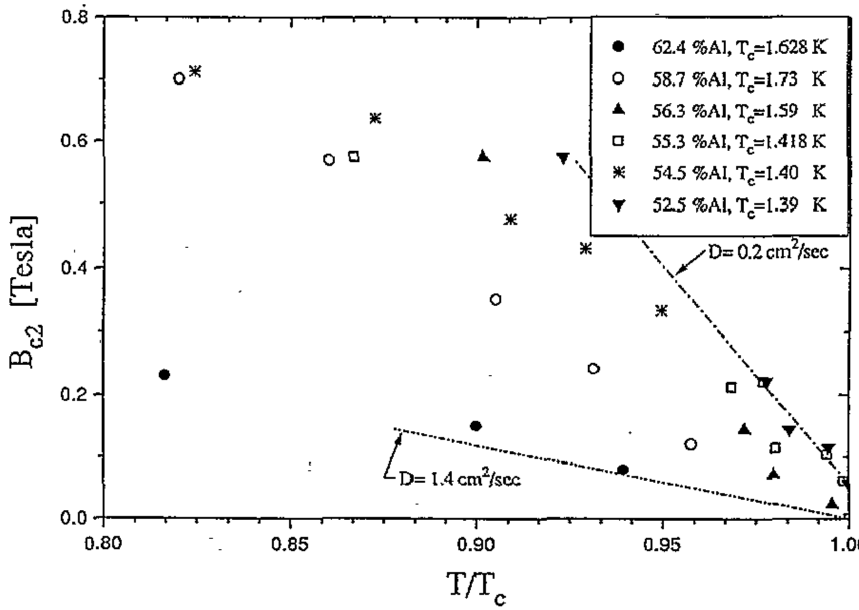

Figure 7. Upper critical-field data versus temperature. The slopes close to T_c determine the magnitude of the diffusion constant for the different films. Note the small values of the diffusion constants.

3. Sample preparation

A series of 21 films of 2000 Å thickness composed of granular Al–Ge were prepared by co-evaporation onto glass substrates held at room temperature. The Al content ranged between

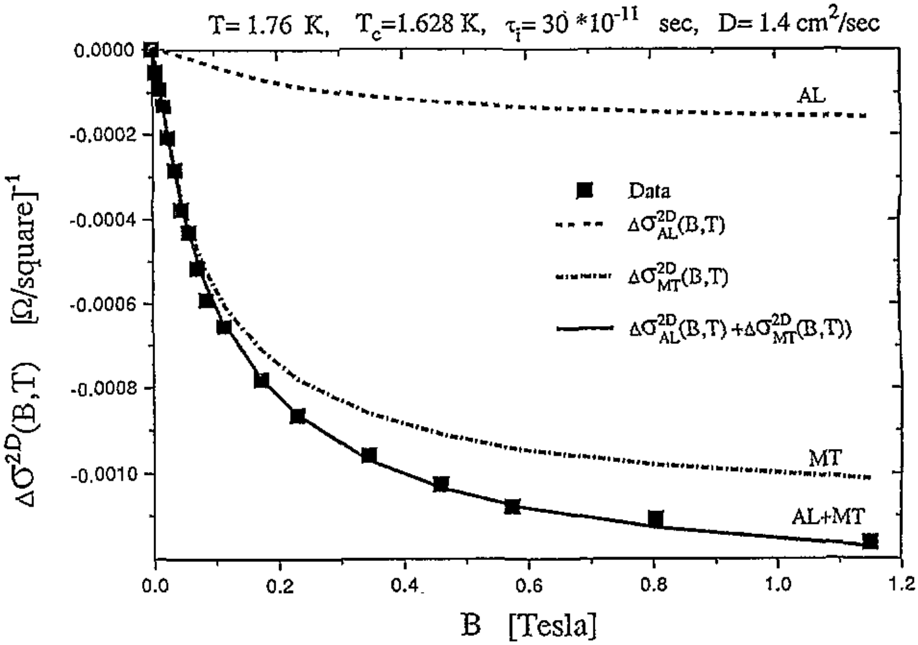

Figure 8. Magnetoconductance data at $T = 1.76$ K versus magnetic field for the most metallic film, the 62.4% Al film. No fitting parameters were used in the Redi and Lopes dos Santos–Abrahams expressions. The diffusion constant of 1.4 cm^2 s^{-1} was obtained from the B_{c2} data of figure 7 and the inelastic scattering time was taken from figure 3. The Aslamazov–Larkin contribution, although small, cannot be neglected in fitting the MC data.

43% and 63% Al for the most insulating to the most metallic film. The metal–insulator transition (MIT) was determined at $\phi_c = 50.7\%$ Al using the unique superconducting properties of the Al clusters in a magnetic field at ^{3}He temperatures. Details of sample fabrication and the determination of ϕ_c can be found in [41] and [42]. Only the metallic films located above the MIT were used in the study of the SCF. Measurements were made in a ^{3}He adsorption cryostat equipped with a small 3.5 T superconducting magnet. Temperatures were measured using a 470 Ω, 0.5 W, Speer carbon resistor that exhibited no measurable magnetoresistance dependence upon magnetic field. Care was taken to minimize temperature gradients between the films and thermometer by minimizing heating to the sample holder and by waiting sufficient time to establish temperature equilibrium. Measuring currents were kept to 1 μA to minimize Joule heating and to ensure that the currents did not destroy the superconductivity.

4. Fitting the experimental results to the theories

An overall view of the 'rounding' of the resistance versus temperature curves is shown in figure 1, for metallic Al–Ge films located above the metal–insulator transition at $\phi = 50.7\%$ Al. The 52.5% Al film closest to ϕ_c had a depressed transition temperature of 1.39 K compared to that of the most metallic film (the 62.4% Al film) of 1.63 K. The 52.5% Al film also had the highest resistance per square of 329 Ω/□ compared to that of the 62.4% film, 17 Ω/□. The most metallic films exhibited relatively sharp transition

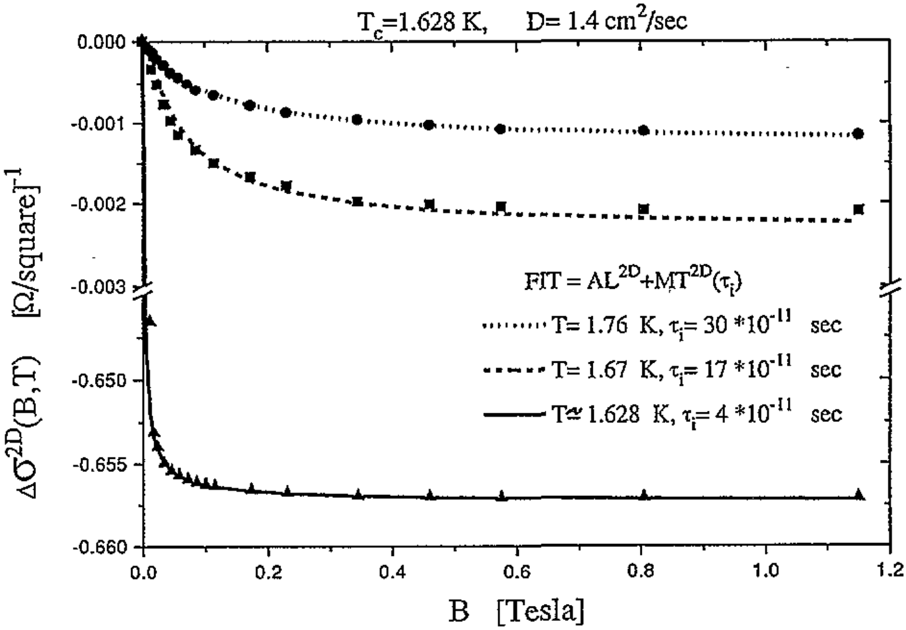

Figure 9. Magnetoconductance data versus magnetic field for three different temperatures of 1.76, 1.67 and 1.628 K for the 62.4% Al film. No adjustable parameters were used in the Redi and Lopes dos Santos–Abrahams expressions. The theories fit the MC data of the 'most' metallic film extremely well.

regions as compared to the films located just above the MIT, which exhibited low-resistance 'tails' prior to going completely superconducting.

We mentioned in the theoretical section that the quantum corrections arising from the weal localization (WL) and electron–electron interaction (EEI) mechanisms must be subtracted out of the zero-field conductivity data. We have measured these two corrections directly by extending the experimental conductivity data of each film to much higher temperatures between 6 and 30 K where the SCF contribution is negligible. This method is illustrated in figure 5 for the most metallic film. In this temperature regime, the resistance increases weakly with decreasing temperature, exhibiting a $-\ln T$ dependence. The prefactor of the $\ln T$ term ranged from 0.000 04 to 0.000 07 for the various films, in excellent agreement with the theory. All of the zero-field excess conductivity data have the WL and EEI corrections removed. The excess conductivity was determined by subtracting the high temperature normal conductivity data extrapolated from the zero-field conductivity data taken at lower temperatures below 6 K.

The excess or SCF conductivity data for the 'most' metallic film having 62.4% Al could be well fitted using the 2D theories of Aslamazov–Larkin (AL) and Maki–Thompson (ML), as shown in figure 6 by the full curve. This film has a T_c of 1.628 K, and a normal resistance of 58 Ω. The film contained 3.4 □ s; thus the resistance per square was 17.3 Ω/□. The T_c values were determined by linearly extrapolating the resistance to zero value in that part of the superconducting transition region that exhibited the steepest slope with respect to temperature. The full curve is derived using equation (2) for the AL contribution and equation (14) for the MT contribution together with equations (13), (18), (19) and (20) to determine the magnitude of the pair-breaking parameter δ as a function of temperature

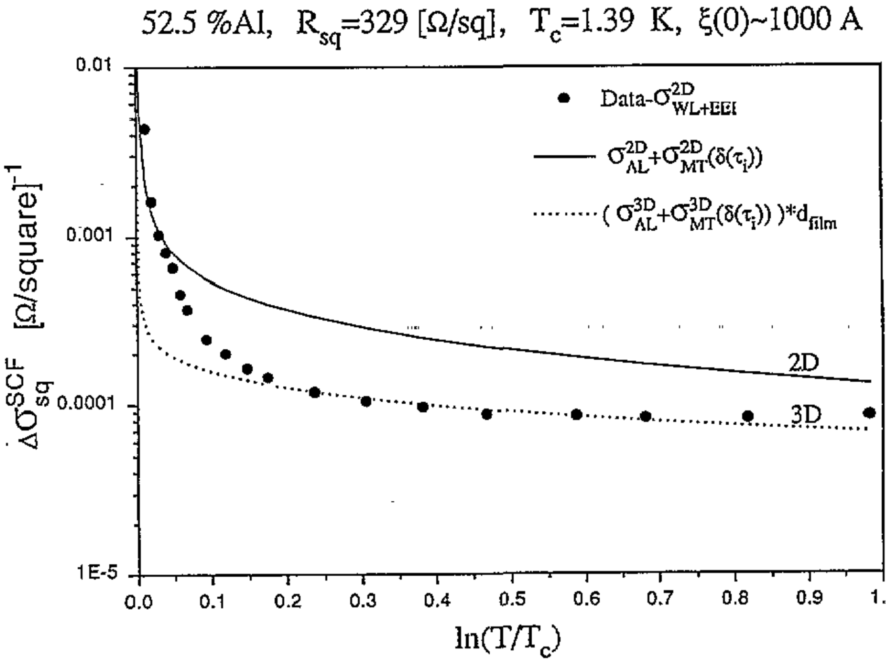

Figure 10. Zero-field SCF data versus temperature for the 'least' metallic film located just above the metal–insulator transition. Note that the 3D theories fit the data well at high temperatures while the 2D theories fit the data better near T_c. A crossover from 3D to 2D behaviour was predicted in figure 2. A value of 1000 Å was used for the fitting parameter $\xi(0)$.

via $\tau_{in}(T)$. No adjustable fitting parameters were used. The fit is excellent, providing experimental support for the validity of the three scattering rate formulae that determine the magnitude of δ.

Values for the inelastic scattering time τ_{in} can also be obtained by fitting the MC theories to the MC data, provided that the diffusion constant D_{dif} is known from an independent measurement. According to equation (11), D_{dif} can be directly extracted from critical-field data taken just below T_c. B_{c2} data taken on our Al–Ge films using the $R_n/2$ criterion are shown in figure 7. R_n is the normal resistance of the film measured in a sufficiently large magnetic field which quenches all superconductivity properties in the film. The magnitudes of D_{dif} ranged from 0.2 to 1.4 cm^2 s^{-1} for the least to most metallic films.

The Redi and Lopes dos Santos–Abrahams MC expressions, equations (7) and (25), were used in fitting the MC theories to the data, using τ_{in} as an adjustable fitting parameter. D_{dif} was fixed at 1.4 cm^2 s^{-1} for the most metallic film. An excellent fit to the MC data taken at $T = 1.76$ K is obtained if τ_{in} takes on a value of 3×10^{-10} s, as demonstrated in figure 8. As can be seen in figure 3, this value falls on the predicted theoretical curve for a film exhibiting 17.3 Ω/□, characterizing this 62.4% Al film. Although the MT term dominates owing to the small value of δ of 0.003, the AL term still contributes a small but important contribution to the total magnitude of the MC.

Fits to the MC data taken at three different temperatures near T_c are shown in figure 9. The inelastic scattering times extracted from the fits are represented by the full circles in figure 3; again the experimental times are in excellent agreement with the theory. The MC data taken at $T \simeq 1.628$ K can also be fitted very well using the asymptotic high-field

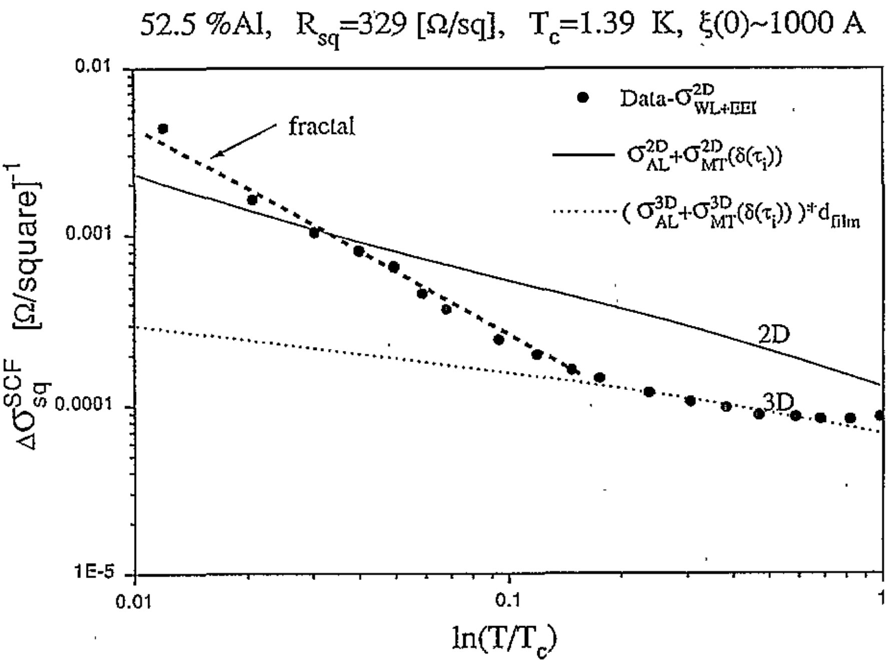

Figure 11. Zero-field SCF data versus temperature now plotted on a log–log plot; these are the same data as in figure 10. Note that the fractal expression given by equation (6) can nicely fit the data close to T_c. A value of 1000 Å was used for the fitting parameter $\xi(0)$.

expression given by equation (28); the fit yields a value of 1.4 cm^2 s^{-1} for the diffusion constant. The weak localization and electron–electron interaction corrections to the MC are negligible compared to the SCF contribution to the MC at these temperatures so close to T_c. Had the MC measurements been taken at much higher temperatures of 4.2 K, the WL contributions to the MC would have to be included also.

Thus for the 'most' metallic films, the 'classical' 2D SCF formulae describe the excess conductivity and magnetoconductivity very well. The MC agreement is somewhat surprising since the superconducting fluctuation scattering time mechanism of Brenig *et al* dominates in the MT term near T_c. As Brenig *et al* point out, τ_{e-fl} has a strong magnetic-field dependence [35]. In large fields this scattering time approaches infinity; and thus this mechanism becomes negligible. We have not included this magnetic field dependence into τ_{e-fl}. Experimentally, the theoretical fits are so good that the field dependence on τ_{e-fl} is not required. Recall that, very close to T_c, the Aslamazov–Larkin term dominates over the Maki–Thompson term and the AL MC term has no dependence upon the scattering times. At least for the MC data close to T_c, we then would predict only a very small correction, had the magnetic field dependence of τ_{e-fl} been included.

We now consider the 'least' metallic film located immediately above the MIT. This film has $R_\square = 329\ \Omega/\square$, a T_c of 1.39 K and an Al content of 52.5%. The zero-field excess conductivity is illustrated in figure 10. The 3D theories, expressed by equations (3) and (15) using $\xi(0) = 1000$ Å, nicely describe the data for temperatures greater than $1.2T_c$ as indicated by the dotted curve in figure 10. The reason for the 3D behaviour, as contrasted to the 2D behaviour observed in the most metallic film, is the shorter superconducting coherence length in this film. The coherence length scales as the square root of the elastic

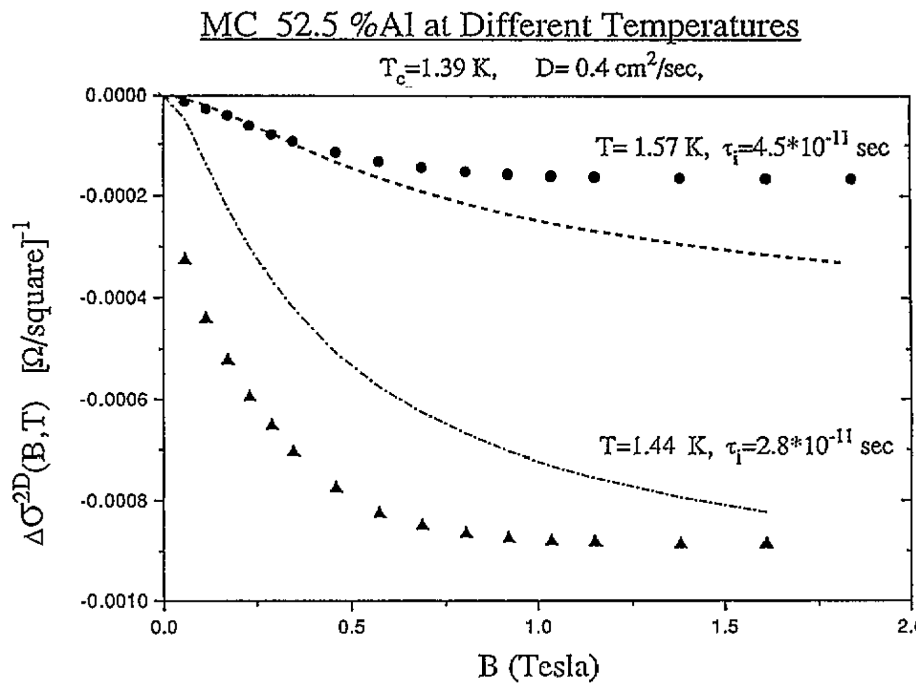

Figure 12. Magnetoconductance data versus magnetic field for the least metallic film at two different measuring temperatures, $T = 1.57$ and 1.44 K. The 2D expressions of Redi and Lopes dos Santos–Abrahams describe these data poorly; the 2D expressions should apply to the data taken at $T = 1.44$ K ($\ln(T/T_c) = 0.04$) where the superconducting coherence length is greater than the film thickness, according to figure 2. This disagreement suggests that an additional conduction process is present, which we associate with Josephson coupling between the Al islands and grains.

mean free path l, which we equate with the small Al grain diameters of roughly 100 Å for this film. Over most of the temperature range, the coherence length is shorter than the film thickness of 2000 Å; a crossover to 2D behaviour is observed for temperatures close to T_c. Although Glover gives an interpolation expression from 3D to 2D for the Aslamazov–Larkin expression, we are not aware of an interpolation formula for the dominating contributions of the Maki–Thompson expressions from 3D to 2D [5]. Thus, we were not able to fit the zero-field data of figure 10 as a continuous function of temperature.

The zero-field excess conductivity data of figure 10 are plotted in a log–log plot in figure 11, where the 3D to 2D crossover is observed with decreasing temperature. It is also possible to characterize the excess conductivity data closest to T_c by a fractal expression given by equation (6) where $\sigma \propto \epsilon^{-x}$ with $x \simeq 1.2$, as indicated by the broken line in figure 11; no Maki–Thompson contribution was included.

Figure 12 shows the MC data for the 'least' metallic film. We had little success fitting the MC data with the Redi and Santos dos Lopes–Abrahams formulae. We used the theoretical inelastic scattering rate predicted by equations (18), (19) and (20) for a film having 329 Ω/□ and varied the diffusion constant as a fitting parameter. For example, consider the data and fit at $T = 1.57$ K in figure 12. The best value for the diffusion constant was 0.4 cm^2 s^{-1} compared to the B_{c2} value of 0.2 cm^2 s^{-1}. However, the fit is poor, which is not surprising since we are trying to fit 2D formulae in a temperature region where the zero-field SCF data suggest that the 3D expressions should be valid; note that the temperature $T = 1.57$ K

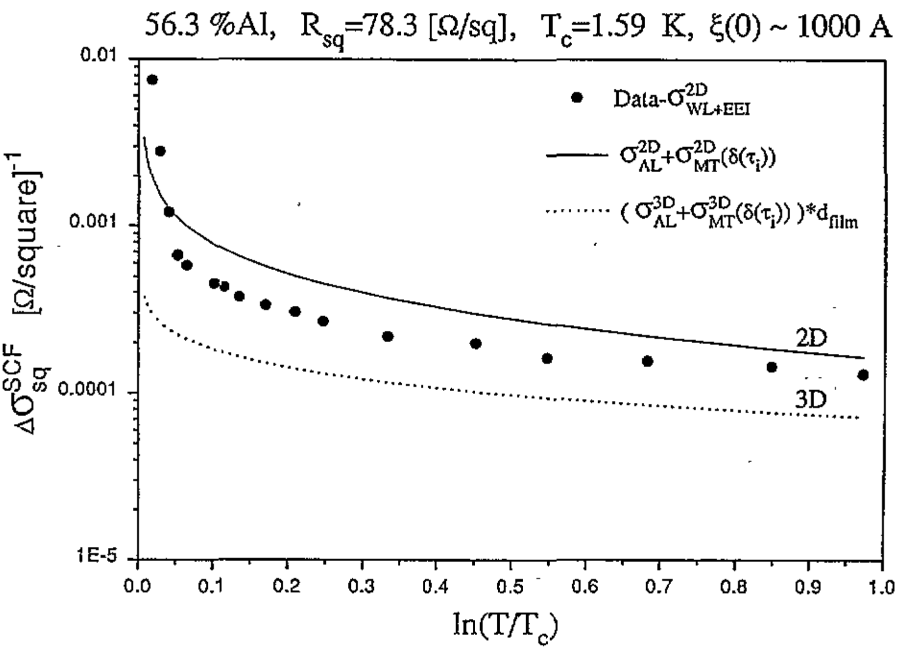

Figure 13. Zero-field SCF data versus temperature for an 'intermediate' metallic film. The data can be described neither by the 2D formulae nor by the 3D formulae. An interpolation formula, particularly for the MT expressions in 3D and 2D, is needed. A value of 1000 Å was used for the fitting parameter $\xi(0)$.

corresponds to a $\ln(T/T_c)$ value of 0.12 in figures 10 and 11. Again, we are not aware of simple 3D formulae that describe the magnetoconductance arising from the AL and MT processes. The fit to the MC data taken at $T = 1.44$ K is particularly bad. It is possible that an additional mechanism is involved in the transition of this least metallic film to the superconducting state. We propose that many of the Al islands and grains behave as Josephson junctions. This would also explain the observed resistance 'tail'. Thus just below and near T_c, the resistance of the film is determined not only by the strength of the superconducting fluctuations within the islands and grains but also by the Josephson coupling between the islands and grains. The most metallic films would have many fewer Josephson junctions, owing to the high Al content that causes metallic contact between most of the islands and grains. Thus this process would be negligible in the most metallic films. We do not know how to treat theoretically the Josephson junction contribution to the conductivity and MC.

We now consider the data taken on an 'intermediate' metallic film. Figure 13 shows the zero-field SCF data for a 56.3% Al film having 78.3 Ω/□ and a T_c of 1.59 K. Surprisingly, the zero-field excess conductivity data can be fitted neither with the 2D expressions (full curve) nor with the 3D expressions (dotted curve) in figure 13. The exception is the single temperature point at $\ln(T/T_c) = 0.04$ where the 2D expression coincides with the data. An interpolation expression between 3D and 2D would probably fit the data well, if such an expression were available for the MT term.

The MC data for this 'intermediate' metallic film are interesting, as shown in figure 14. By chance, one of the measuring temperatures chosen was 1.657 K, which corresponds to $\ln(T/T_c) = 0.04$, exactly where the 2D formulae describe the zero-field data. The

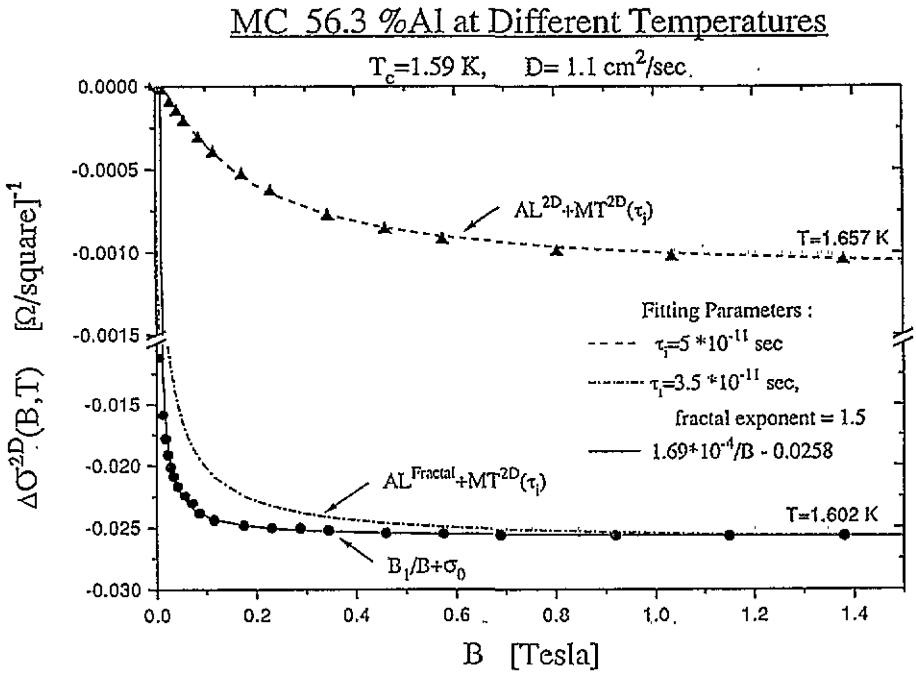

Figure 14. Magnetoconductance data versus magnetic field for the 'intermediate' metallic film. The 2D formulae of Redi and Lopes dos Santos–Abrahams describe the $T = 1.657$ K data very well. Refer to the text for details on fitting the $T = 1.602$ K data.

2D formulae of Redi and Lopes dos Santos–Abrahams fit the MC data very well at this temperature using the measured diffusion constant D_{dif} of 1.1 cm^2 s^{-1} from the B_{2c} data and the theoretical inelastic time predicted from equations (18), (19) and (20).

The fit to the MC data at $T = 1.602$ K was much more problematic. Here the measuring temperature was very close to T_c with $\ln(T/T_c) = 0.008$. From figure 13, we observe that the 2D expressions underestimate the zero-field excess conductivity data at this temperature, being 10 times smaller. However, we can fit the zero-field data nicely using the fractal expression, equation (6), with x set to 1.5. If we then try to fit the MC data at $T = 1.602$ K, correcting the prefactor of the Redi expression with equation (6) rather than equation (2), then we obtain the fit indicated by the dash-dotted curve in figure 14. Using the 'fractal' prefactor expression ensures that the theoretical MC fit saturates at the correct value at high fields. The fit with this correction is acceptable. Interestingly, if one uses the high-field asymptotic expression (28), an outstandingly good fit is obtained as indicated by the full curve in figure 14. The only fitting parameter that appears in the high-field asymptotic expression is the diffusion constant, whose value was chosen to be 1.1 cm^2 s^{-1} taken from the B_{c2} data.

5. Conclusions

Initial attempts to fit the zero-field SCF data were completely unsuccessful until we realized that quantum corrections from weak localization and electron–electron interactions effects must be subtracted out from the zero-field data. We strongly recommend extending the zero-field measurements up to temperatures as high as $20T_c$ in order to determine experimentally the magnitudes of these two quantum corrections. The corrected zero-field excess conductivity data and the MC data near T_c could be fitted extremely well to the data of the most metallic films using the well know Aslamazov–Larkin and Maki-Thompson theories, and using a temperature-dependent pair-breaking parameter determined from theoretical scattering rates.

In contrast, the film located nearest to the metal–insulating transition exhibited anomalous magnetoconductance behaviour that could not be fitted by these theories. We propose that Josephson coupling between the Al islands and grains complicates the conduction processes in these films located near the MIT.

Acknowledgments

We greatly thank Dr Yoad Yagil for sample preparation and are grateful to Dr M Witcomb for the EDAX (energy-dispersive x-ray analysis) measurements made at the University of Witwatersrand. We thank both Daniel Racah for valuable discussions and Yinon Ashkenazy for his B_{c2} data. We are grateful to Anne-Marie Arav for editing. We acknowledge the Sackler family and the German–Israeli Foundation for Scientific Research and Development (GIF) for financial support.

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